# Handling partially ordered preferences in possibilistic logic - A survey discussion -

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**Abstract.** This paper advocates possibilistic logic with partially ordered priority weights as a powerful representation format for handling preferences. An important benefit of such a logical setting is the ability to check the consistency of the specified preferences. We recall how Qualitative Choice Logic statements (and related ones), as well as CP-nets preferences can be represented in this framework. We investigate how a generalization of CP-nets, namely CP-theories, can also be handled in a partially ordered possibilistic logic setting. Finally we suggest how this framework may be used for handling preference queries.

## 1 INTRODUCTION

Possibilistic propositional logic is a logic where classical propositions are associated with priority levels; see [23] for an introduction. In this setting, inconsistency amounts to having a classically inconsistent set of propositions that are all associated with strictly positive priority levels. In particular, one cannot give priority both to $p$ and to $\neg p$. Possibilistic logic may be used for handling uncertainty, or preferences. In this discussion paper, we survey the use of possibilistic logic for representing preferences, and compare it with popular representation settings for preferences such as CP-nets [14], CP-theories [36], or Qualitative Choice Logic [15]; see [17] for an introductory survey on the handling of preferences in artificial intelligence, operations research, or data bases literature.

After a brief refresher on possibilistic logic, the paper provides an account of the handling of ordered conjunctions and disjunctions for preference modeling in possibilistic logic. We then advocate the use of partially ordered symbolic weights for coping with the need of leaving room for incomparability, as observed in CP-nets or in CP-theories settings.

## 2 POSSIBILISTIC LOGIC

We consider a propositional language where formulas are denoted by $p_1$, ..., $p_n$, and $\Omega$ is its set of interpretations. Let $B^N = \{(p_j, \alpha_j) \mid j = 1, \ldots, m\}$ be a possibilistic logic base where $p_j$ is a propositional logic formula and $\alpha_j \in \mathcal{L} \subseteq [0, 1]$ is a priority level [23]. The logical conjunctions and disjunctions are denoted $\wedge$ and $\vee$. Each formula $(p_j, \alpha_j)$ means that $N(p_j) \geq \alpha_j$, where $N$ is a necessity measure, i.e., a set function satisfying the property $N(p \wedge q) = \min(N(p), N(q))$. A necessity measure is associated to a possibility distribution $\pi$ as follows:

$N(p) = \min_{\omega \notin M(p)}(1 - \pi(\omega)) = 1 - \Pi(\neg p)$, where $\Pi$ is the possibility measure associated to $N$ and $M(p)$ is the set of models induced by the underlying propositional language for which $p$ is true.

The base $B^N$ is associated to the possibility distribution $\pi^N_B(\omega) = \min_{j=1,\ldots,m} \pi_{(p_j,\alpha_j)}(\omega)$ on the set of interpretations, where $\pi_{(p_j,\alpha_j)}(\omega) = 1$ if $\omega \in M(p_j)$, and $\pi_{(p_j,\alpha_j)}(\omega) = 1 - \alpha_j$ if $\omega \notin M(p_j)$. An interpretation $\omega$ is all the more possible as it does not violate any formula $p_j$ having a higher priority level $\alpha_j$. Hence, this possibility distribution is expressed as a min-max combination:

$$\pi^N_B(\omega) = \min_{j=1,\ldots,m} \max(1 - \alpha_j, I_{M(p_j)}(\omega))$$

where $I_{M(p_j)}$ is the characteristic function of $M(p_j)$. So, if $\omega \notin M(p_j)$, $\pi^N_B(\omega) \leq 1 - \alpha_j$, and if $\omega \in \bigcap_{j \in J} M(\neg p_j)$, $\pi^N_B(\omega) \leq \min_{j \in J}(1 - \alpha_j)$. It is a description "from above" of $\pi^N_B$, which is the least specific possibility distribution in agreement with the knowledge base $B^N$. A possibilistic base $B^N$ can be transformed in a base where the formulas $p_i$ are clauses (without altering the distribution $\pi^N_B$). We can still see $B^N$ as a conjunction of weighted clauses, i.e., as an extension of the conjunctive normal form.

A dual representation of the possibilistic logic is based on guaranteed possibility measures. A guaranteed possibility measure is associated to a possibility distribution $\pi$ as follows: $\Delta(p) = \min_{\omega \in M(p)} \pi(\omega)$. Hence a logical formula is a pair $[q, \beta]$, interpreted as the constraint $\Delta(q) \geq \beta$, where $\Delta$ is a guaranteed possibility (anti-)measure characterized by $\Delta(p \vee q) = \min(\Delta(p), \Delta(q))$ and $\Delta(\emptyset) = 1$. In such a context, a base $B^\Delta = \{[q_i, \beta_i] \mid i = 1, \ldots, n\}$ is associated to the distribution

$$\pi^\Delta_B(\omega) = \max_{i=1,\ldots,n} \pi_{[q_i,\beta_i]}(\omega)$$

with $\pi_{[q_i,\beta_i]}(\omega) = \beta_i$ if $\omega \in M(q_i)$ and $\pi_{[q_i,\beta_i]}(\omega) = 0$ otherwise. If $\omega \in M(q_i)$, $\pi^\Delta_B(\omega) \geq \beta_i$, and if $\omega \in \bigcup_{i \in I} M(q_i)$, $\pi^\Delta_B(\omega) \geq \max_{i \in I} \beta_i$. So this base is a description "from below" of $\pi^\Delta_B$, which is the most specific possibility distribution in agreement with the knowledge base $B^\Delta$. A dual possibilistic base $B^\Delta$ can always be transformed in a base in which the formulas $q_j$ are conjunctions of literals (cubes) without altering $\pi^\Delta_B$.

A possibilistic logic base $B^\Delta$ expressed in terms of guaranteed possibility measure can always be rewritten equivalently in terms of standard possibilistic logic $B^N$ based on necessity measures [10, 8] and conversely with the equality $\pi^N_B = \pi^\Delta_B$. This transformation is similar to a description from below of $\pi^N_B$.

In case of mutually exclusive propositions, $p_1$,... $p_i$, ..., $p_n$, if $N(p_1) \geq \alpha_1 > 0$, then $N(p_2) = ... = N(p_n) = 0$ for the sake

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of consistency. But, the set of requirements $\Delta(p_1) \geq \beta_1 > 0$, ..., $\Delta(p_i) \geq \beta_i > 0$, ..., $\Delta(p_n) \geq \beta_n > 0$ is consistent, and if $\beta_1 = 1 > \cdots > \beta_i > \cdots > \beta_n > 0$, it can be equivalently represented by $N(p_1) \geq \alpha_1$, $N(p_1 \vee p_2) \geq \alpha_2$, ..., $N(p_1 \vee p_2 \vee \cdots \vee p_i) \geq \alpha_i$, ..., $N(p_1 \vee p_2 \vee \cdots \vee p_n) \geq \alpha_n$, with $\alpha_i = 1 - \beta_{i+1}$ and $\beta_{n+1} = 0$.

What makes the possibilistic logic setting particularly appealing for the representation of preferences is not only the fact that the language incorporates priority levels explicitly, but the existence of different representation formats [9, 21], whose representation power is equivalent [4, 5], but which are more or less natural or suitable for expressing preferences. Thus, preferences can be represented

- as prioritized goals, i.e. possibilistic formulas of the form $(p_i, \alpha_i)$ meaning that $N(p_i) \geq \alpha_i$, and stating that making $p_i$ true has priority level $\alpha_i$;
- in terms of guaranteed satisfaction levels by means of formulas of the form $[q_j, \beta_j]$ understood as $\Delta(q_j) \geq \beta_j$, and stating that as soon as one satisfies $q_j$ then one reaches at least satisfaction levels $\beta_j$ [6];
- by means of a possibility distribution, where an ordering is explicitly stated between the interpretations of the language; the ordering is complete as soon as the values of the possibility degrees are known;
- in terms of conditionals of the form $\Pi(p \wedge q) > \Pi(p \wedge \neg q)$ (including the case where is $p$ is a tautology, i.e., $N(q) > N(\neg q) = 0 \Leftrightarrow \Pi(q) = 1 > \Pi(\neg q)$) expressing that in the context where $p$ is true, there is at least one interpretation where $q$ is true which is preferred to all interpretations where $q$ is false. As pointed out in [18, 20] and analyzed in details in [33], there are other comparative statements of interest, namely $\Delta(p \wedge q) > \Pi(p \wedge \neg q)$, $\Pi(p \wedge q) > \Delta(p \wedge \neg q)$, and $\Delta(p \wedge q) > \Delta(p \wedge \neg q)$. For instance, the first one of the three is clearly more drastic than the initial one we considered since it requires that in context $p$, *any* interpretation where $q$ is true is preferred to all interpretations where $q$ is false;
- as Bayesian-like networks, since a possibilistic logic base can be encoded either as a qualitative or as a quantitative possibilistic networks and vice-versa. Qualitative and quantitative possibilistic networks are respectively associated with a minimum- and a product-based definition of conditioning [3].

## 3 ORDERED CONJUNCTIONS AND DISJUNCTIONS

In the following, propositional variables refer to properties of items, to be rank-ordred in terms of preferences, and formulas represent requests to be satisfied.

**Conjunctions**. Putting priorities on goals is easy to understand as a way for specifying preferences, and amounts to express a *weighted conjunction* of goals, which may be stated by means of '*and if possible*' in statements such as "$p_1$ and if possible $p_2$ and if possible $p_3$" ($p_1$ is more important than $p_2$, which is itself more important than $p_3$). Such statements have been first considered in [34] in another setting.

The $p_i$'s may be logically independent or not. For the sake of simplicity, we use here three conditions only, but what follows would straightforwardly extend to $n$ conditions. We denote by $M(p_i)$, $M(p_i \wedge p_j)$, the set of items (if any) satisfying condition $p_i$, the set of items (if any) satisfying $p_i$ and $p_j$, and so on. So the query "$p_1$ is required *and if possible* $p_2$ also *and if possible* $p_3$ too", has the following intended meaning ($\gg$ reads "is preferred to")

$$M(p_1 \wedge p_2 \wedge p_3) \gg M(p_1 \wedge p_2 \wedge \neg p_3) \gg M(p_1 \wedge \neg p_2) \gg M(\neg p_1)$$

i.e., one prefers to have the three conditions satisfied rather than the two first ones only, which is itself better than having just the first condition satisfied (which in turn is better than not having even the first condition satisfied). This is indeed simply described in possibilistic logic as the conjunction of prioritized goals $\mathcal{C} = \{(p_1, \gamma_1), (p_2, \gamma_2), (p_3, \gamma_3)\}$ with $1 = \gamma_1 > \gamma_2 > \gamma_3 > 0$. It can be checked that this possibilistic logic base is associated with the possibility distribution

$$\begin{aligned} \pi_{\mathcal{C}}(\omega) = {} & 1 \text{ if } \omega \in M(p_1 \wedge p_2 \wedge p_3) \\ & 1 - \gamma_3 \text{ if } \omega \in M(p_1 \wedge p_2 \wedge \neg p_3) \\ & 1 - \gamma_2 \text{ if } \omega \in M(p_1 \wedge \neg p_2) \\ & 0 \text{ if } \omega \in M(\neg p_1). \end{aligned}$$

which fully agrees with the above ordering.

Moreover in a logical encoding, a query such as "find the $x$'s such that condition $Q$ is true", i.e., $\exists x\ Q(x)$? is usually processed by refutation. Using a small old trick due to Green [27], it amounts to adding the formula(s) corresponding to $\neg Q(x) \vee answer(x)$, expressing that if item $x$ satisfies condition $Q$ it belongs to the answer, to the logical base describing the content of the database. It enables theorem-proving by resolution to be applied to question-answering. This idea extends to preference queries expressed in a possibilistic logic setting [13]. The expression of the query $\mathcal{Q}$ corresponding to the above set of prioritized goals is then of the form

$$\begin{aligned} \mathcal{Q} = \{ & (\neg p_1(x) \vee \neg p_2(x) \vee \neg p_3(x) \vee answer(x), 1), \\ & (\neg p_1(x) \vee \neg p_2(x) \vee answer(x), 1 - \gamma_3), \\ & (\neg p_1(x) \vee answer(x), 1 - \gamma_2)\}. \end{aligned}$$

where $1 > 1 - \gamma_3 > 1 - \gamma_2$. Then, the levels associated with the possibilistic logic formulas expressing the preference query are directly associated with the possibility levels of the possibility distribution $\pi_{\mathcal{C}}$ providing its semantics.

**Disjunctions**. We may also consider *disjunctive* queries with priorities, i.e., queries of the form "$p_1$ is required with priority, or failing this $p_2$, or still failing this $p_3$", as discussed in [13]. It has the following intended meaning in terms of interpretations:

$$M(p_1) \gg M(\neg p_1 \wedge p_2) \gg M(\neg p_1 \wedge \neg p_2 \wedge p_3) \gg M(\neg p_1 \wedge \neg p_2 \wedge \neg p_3).$$

As can be checked, it corresponds to the following possibilistic logic base representing a *conjunction* of prioritized goals:

$$\mathcal{D}_N = \{(p_1 \vee p_2 \vee p_3, 1), (p_1 \vee p_2, \gamma_2), (p_1, \gamma_3)\}.$$

(with $1 > \gamma_2 > \gamma_3$) whose associated possibility distribution is

$$\begin{aligned} \pi_{\mathcal{D}_N}(\omega) = {} & 1 \text{ if } \omega \in M(p_1) \\ & 1 - \gamma_3 \text{ if } \omega \in M(\neg p_1 \wedge p_2) \\ & 1 - \gamma_2 \text{ if } \omega \in M(\neg p_1 \wedge \neg p_2 \wedge p_3) \\ & 0 \text{ if } \omega \in M(\neg p_1 \wedge \neg p_2 \wedge \neg p_3), \end{aligned}$$

which is clearly in agreement with the above ordering. It can be also equivalently expressed in a question-answering perspective by the possibilistic logic base:

$$\begin{aligned} \mathcal{Q}' = \{ & (\neg p_1(x) \vee answer(x), 1), \\ & (\neg p_2(x) \vee answer(x), 1 - \gamma_3), \\ & (\neg p_3(x) \vee answer(x), 1 - \gamma_2)\}. \end{aligned}$$

which states that if an item $x$ satisfies $p_1$, then it belongs to the answer to degree 1, and if it satisfies $p_2$ (resp. $p_3$), then it belongs to the answer to a degree at least equal to $1-\gamma_3$ (resp $1-\gamma_2$).

As noticed in [13, 24], there is a perfect duality between conjunctive and disjunctive queries. Indeed the disjunctive query "$p_3$ is required, or better $p_2$, or still better $p_1$" can be also equivalently expressed under the conjunctive form "$p_1$ or $p_2$ or $p_3$ is required *and if possible* $p_1$ or $p_2$, *and if possible* $p_1$". Conversely, the conjunctive query "$p_1$ is required *and if possible* $p_2$ *and if possible* $p_3$" can be equivalently stated as the disjunctive query "$p_1$ is required, or better $p_1$ and $p_2$, or still better $p_1$ and $p_2$ and $p_3$". This can be checked on their respective possibilistic logic representations.

Let us point out the close relation between the possibilistic representation and qualitative choice logic (QCL) [15]. Indeed QCL introduces a new connective denoted $\times$, where $p_1 \times p_2$ means "if possible $p_1$, but if $p_1$ is impossible then (at least) $p_2$". This corresponds to a disjunctive preference of the above type. Then, the query "$p_1$, *or at least* $p_2$, *or at least* $p_3$", which, as already explained, corresponds to stating that $p_1$ is fully satisfactory, $p_2$ instead is less satisfactory, and $p_3$ instead is still less satisfactory, can be directly represented in the possibilistic logic based on guaranteed possibility measures [2]. Using the notation of Section 2, the corresponding weighted base simply writes $\mathcal{D}_\Delta = \{[p_1, 1], [p_2, 1-\gamma_3], [p_3, 1-\gamma_2]\}$, which clearly echoes $\mathcal{Q}'$. It encodes the same possibility distribution on models as the necessity-based possibilistic logic base $\mathcal{D}_N$.

Note that in $\mathcal{Q}'$, as in $\mathcal{Q}$, the weights of the possibilistic logic formulas express a priority among the answers $x$ that may be obtained. They may be also viewed as representing the levels of satisfaction of the answers obtained.

The linguistic expression of conjunctive queries may suggest that $p_1$, $p_2$, $p_3$ are logically independent conditions that one would like to cumulate, as in the query "I am looking for a reasonably priced hotel, *if possible* downtown, *and if possible* not far from the station", while in disjunctive queries one may think of $p_3$ as a relaxation of $p_2$, itself a relaxation of $p_1$. In fact there is no implicit limitation on the type of conditions involved in conjunctive or disjunctive queries. For instance, a conjunctive query such as "I am looking for a hotel less than 2 km from the beach, *if possible* less than 1 km from the beach, *and if possible* on the beach", corresponds to the idea of approximating a fuzzy requirement, such as "close to the beach" by three of its level cuts, which are then relaxation or strengthening of one another.

**Hybrid queries**. A mutual refinement of the two above types of queries leads to "full discrimination-based queries" [13]. It amounts to computing a lexicographic ordering of the different worlds (here $2^3 = 8$ with 3 conditions), under the tacit, default assumption that it is always better to have a condition fulfilled rather than not, even if a more important condition is not satisfied. However, it is clear that sometimes satisfying an auxiliary condition while failing to satisfy the main condition may be of no interest, as in the example "I would like a coffee if possible with sugar", where having sugar or not, if no coffee is available, makes no difference. There are even situations, in case of a conditional preference, where it may be worse to have $p_2$ satisfied than not when $p_1$ cannot be satisfied, as in the example "I would like a Ford car if possible black" (if one prefers any other color for non Ford cars). Full discrimination-based queries are thus associated with the following preference ordering:

$$M(p_1 \wedge p_2 \wedge p_3) \gg M(p_1 \wedge p_2 \wedge \neg p_3) \gg M(p_1 \wedge \neg p_2 \wedge p_3) \gg M(p_1 \wedge \neg p_2 \wedge \neg p_3) \gg M(\neg p_1 \wedge p_2 \wedge p_3) \gg M(\neg p_1 \wedge p_2 \wedge \neg p_3) \gg M(\neg p_1 \wedge \neg p_2 \wedge p_3) \gg M(\neg p_1 \wedge \neg p_2 \wedge \neg p_3)$$

It can be checked that it can be encoded in possibilistic logic under the form (we only give the question-answering form here):

$$\mathcal{Q}" = \{(\neg p_1(x) \vee \neg p_2(x) \vee \neg p_3(x) \vee answer(x),\ 1), (\neg p_1(x) \vee \neg p_2(x) \vee answer(x),\ \alpha), (\neg p_1(x) \vee \neg p_3(x) \vee answer(x),\ \alpha'), (\neg p_1(x) \vee answer(x),\ \alpha''), (\neg p_2(x) \vee \neg p_3(x) \vee answer(x),\ \beta), (\neg p_2(x) \vee answer(x),\ \beta'), (\neg p_3(x) \vee answer(x),\ \gamma)\}$$

with $1 > \alpha > \alpha' > \alpha'' > \beta > \beta' > \gamma$.

**Constraints and wishes**. A request of the form "$\mathcal{A}$ *and if possible* $\mathcal{B}$", where both $\mathcal{A}$ and $\mathcal{B}$ are prioritized *sets* of specifications may be understood in fact in different ways. Either we consider that $\mathcal{A}$ and $\mathcal{B}$ are of the same nature, and the request may be reorganized into a unique set of prioritized goals, or alternatively one may consider that what is expressed in $\mathcal{B}$ is not at all compulsory, but are just "wishes" that should be used for further discrimination between situations that would be ranked in the same way according to $\mathcal{A}$ [22, 24]. We are going to examine the difference between the two points of view, in the simple case where both $\mathcal{A}$ and $\mathcal{B}$ are made of two conditions, namely

$\mathcal{A} = \{(a_2, 1), (a_1, 1-\alpha)\}$ with $1 > 1-\alpha > 0$, and
$\mathcal{B} = \{(b_2, 1), (b_1, 1-\alpha')\}$ with $1 > 1-\alpha' > 0$.

We further assume in this example that i) the conditions in $\mathcal{A}$ are nested, as well as the ones in $\mathcal{B}$, and ii) the conditions in $\mathcal{B}$ are *refinements* of those in $\mathcal{A}$, which is necessary for allowing for a "wish" understanding of $\mathcal{B}$ [22] in the second view. This means that we assume $M(a_2) \supseteq M(a_1) \supseteq M(b_1)$, $M(a_2) \supseteq M(b_2) \supseteq M(b_1)$ and $\alpha' < \alpha$, with $M(b_2) \bigcap M(a_2) \neq \emptyset$.

When both $\mathcal{A}$ and $\mathcal{B}$ are viewed as *constraints*, i.e. as sets of prioritized goals, namely and respectively, the request "$\mathcal{A}$ *and if possible* $\mathcal{B}$" translates into a *unique* set $\mathcal{G}$ of prioritized goals, where the goals in $\mathcal{B}$ are discounted by $1-\lambda$, where $\alpha < \lambda$ so that the weakest constraint in $\mathcal{A}$ has priority over the strongest constraint in $\mathcal{B}$:

$$\mathcal{G} = \{(a_2, 1), (a_1, 1-\alpha), (b_2, \min(1, 1-\lambda)), (b_1, \min(1-\alpha', 1-\lambda))\}.$$

This possibilistic logic base is associated with the possibility distribution

$$\pi_{\mathcal{G}}(\omega) = \begin{cases} 1 \text{ if } \omega \in M(a_1 \wedge b_1) \\ \lambda \text{ if } \omega \in M(a_1 \wedge \neg b_1) \\ \alpha \text{ if } \omega \in M(a_2 \wedge \neg a_1 \wedge b_2) \\ 0 \text{ if } \omega \in M(\neg a_2). \end{cases}$$

Let us now consider the second view where only $\mathcal{A}$ is regarded as a set of prioritized constraints, while $\mathcal{B}$ is a set of *prioritized wishes*. Now we keep $\mathcal{A}$ and $\mathcal{B}$ separate. Each interpretation $\omega$ is the associated with a pair of values: the first (resp. the second) value is equal to $1-\gamma^*$ (resp. $1-\delta^*$) where $\gamma^*$ (resp. $\delta^*$) is the priority of the formula violated by $\omega$ having the highest priority in $\mathcal{A}$ (resp. $\mathcal{B}$). We obtain, the following *vector-valued* possibility distribution:

$$\pi_{(\mathcal{A},\mathcal{B})}(\omega) = (1,1) \text{ if } \omega \in M(a_1 \wedge b_1)$$
$$(1,\alpha') \text{ if } \omega \in M(a_1 \wedge \neg b_1 \wedge b_2)$$
$$(1,0) \text{ if f } \omega \in M(a_1 \wedge \neg b_2)$$
$$(\alpha,\alpha') \text{ if } \omega \in M(a_2 \wedge \neg a_1 \wedge b_2)$$
$$(\alpha,0) \text{ if } \omega \in M(a_2 \wedge \neg a_1 \wedge \neg b_2)$$
$$(0,0) \text{ if } \omega \in M(\neg a_2).$$

Note the lexicographic ordering of the evaluation vectors. We now have 6 layers of interpretations (instead of 4 in the previous view), which makes it clear that this second view is more refined. However, in the rest of the paper, all the preferences are viewed as constraints.

## 4 CP-NETS IN POSSIBILISTIC LOGIC

This section presents a possibilistic logic approach *with symbolic weights* that generalizes the representation of preferences reviewed in Section 3. The proposed method enables the handling of conditional preferences, as well as the representation of prioritized conjunctions. The approach is both more faithful to user's preferences than the CP-net approach as we shall see. Formally, a CP-net [28] $N$ over the set of Boolean variables $V = \{X_1, \cdots, X_n\}$ is a directed graph over the nodes $X_1, \cdots, X_n$, and there is a directed edge from $X_i$ to $X_j$ if the preference over the value $X_j$ is conditioned on the value of $X_i$. Each node $X_i \in V$ is associated with a conditional preference table $CPT(X_i)$ that associates a strict (possibly empty) partial order $>_{CP} (u_i)$ with each possible instantiation $u_i$ of the parents of $X_i$. A complete preference ordering satisfies a CP-net $N$ iff it satisfies each conditional preference expressed in $N$. In this case, the preference ordering is said to be consistent with $N$. Since CP-nets encode partial orders, while possibilistic logic encodes a complete preorder (when priorities are given), these two formalisms cannot be equivalent. The best we can do is to approximate CP-nets in possibilistic logic. A faithful approximation of a CP-net in possibilistic logic consists in preserving all strict preferences induced by the CP-net [18, 20]. However, by enforcing appropriate ordering constraints between symbolic weights, we can obtain an exact representation of a CP-net in possibilistic logic with symbolic weights [29, 32], as explained now.

Using an example, we first present the idea of representing conditional preferences by means of possibilistic logic formulas with symbolic weights. We then introduce a natural preorder between formulas, which may be then completed by further constraints between symbolic weights. Lastly, a general evaluation procedure is outlined.

### 4.1 Possibilistic representation of conditional preferences – An example.

Example 1 taken from [36], is about planning holidays, where one has the following preferences: one can either go next week ($n$) or later in the year ($\bar{n}$). One can decide to go either to Oxford ($o$) or to Manchester ($\bar{o}$), and one can either take a plane ($p$) or drive and take a car ($\bar{p}$). So, there are three variables $X_1, X_2$ and $X_3$ where $\underline{X_1}$=$\{n, \bar{n}\}$, $\underline{X_2}$=$\{o, \bar{o}\}$ and $\underline{X_3}$=$\{p, \bar{p}\}$, where $\underline{X}$ stands for a set of possible assignments of X. Suppose the person prefers to go next week than later in the year and prefers to fly than to drive unless he goes later in the year to Manchester.

Such preferences can be encoded as prioritized goals in possibilistic logic, as explained now. The possibilistic encoding of the conditional preference "in context $c$, $a$ is preferred to $b$" is a pair of possibilistic formulas: $\{(\neg c \vee a \vee b, 1), (\neg c \vee a, \alpha)\}$ with $1 > \alpha > 0$.

Namely if $c$ is true, one should have $a$ or $b$ (the choice is only between $a$ and $b$), and in context $c$, it is somewhat imperative to have $a$ true. This encodes a constraint of the form $N(\neg c \vee a) \geq \alpha$, itself equivalent here to a constraint on a conditional necessity measure $N(a|c) \geq \alpha$ (see, e.g., [23]). This is still equivalent to $\Pi(\neg a|c) \leq 1 - \alpha$, where $\Pi$ is the dual possibility measure associated with $N$. It expresses that the possibility of not having $a$ is upper bounded by $\alpha$, i. e. $\neg a$ is all the more impossible as $\alpha$ is small. Such a modeling has been proposed in [30] for representing preferences, and approximating CP-nets. It can be proved that $\{(\neg c \vee a \vee b, 1), (\neg c \vee a, \alpha)\}$ is equivalent to requesting $N(a|c) \geq \alpha > 0 = N(b|c)$. Note that when $b \equiv \neg a$, the first clause becomes a tautology, and thus does not need to be written. Strictly speaking, the possibilistic clause $(\neg c \vee a, \alpha)$ expresses a preference for $a$ (over $\neg a$) in context $c$. The clause $(\neg c \vee a \vee b, 1)$ is only needed if $a \vee b$ does not cover all the possible choices. Assume $a \vee b \equiv \neg d$ (where $\neg d$ is not a tautology), then it makes sense to understand the preference for $a$ over $b$ in context $c$, as the fact that in context $c$, $b$ is a default choice if $a$ is not available. If one wants to open the door to remaining choices, it is always possible to use $(\neg c \vee a \vee b, \alpha')$ with $\alpha' > \alpha$, instead of $(\neg c \vee a \vee b, 1)$. Thus, the approach easily extends to non binary choices. For instance, "I prefer Renault ($r$) to Chrysler ($c$) and Chrysler to Ford ($f$)" is encoded as $\{(r \vee c \vee f, 1), (r \vee c, \alpha), (r, \alpha')\}$, with $\alpha > \alpha'$.

It is worth noticing that the encoding of preferences in this framework also applies to Lacroix and Lavincy's approach [34], namely, when one wants to express that "$p_1 \wedge p_2$ is preferred to $p_1 \wedge \neg p_2$" and $p_1$ is mandatory. It is encoded by $((p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2), 1)$, equivalent to $(p_1, 1)$, and by $(p_1 \wedge p_2, 1-\alpha)$ equivalent to $(p_1, 1-\alpha)$ and $(p_2, 1-\alpha)$, $(p_1, 1-\alpha)$ being subsumed by $(p_1, 1)$. Thus, one retrieves the encoding $(p_1, 1)$ and $(p_2, 1-\alpha)$, already proposed in Section 3.

### 4.2 Preorder induced by formulas with symbolic priority levels.

When one does not know precisely how imperative the preferences are, the weights can be handled in a symbolic manner, and then partially ordered. This means that the weights are replaced by variables that are assumed to belong to a linearly ordered scale (the strict order will be denoted by $\succ$ on this scale), with a top element (denoted 1) and a bottom element (denoted 0). Thus, $1 - (.)$ should be regarded here just as denoting an order-reversing map on this scale (without having a numerical flavor necessarily), with $1 - (0) = 1$, and $1 - (1) = 0$. On this scale, one has $1 \succ 1 - \alpha$, as soon as $\alpha \neq 0$. The weights are different from 1 but are all greater than 0. We assume that the order-reversing map relates to two scales: the one graded in terms of necessity degrees, or if we prefer here in terms of imperativeness, and the one graded in terms of possibility degrees, i.e. here, in terms of satisfaction levels. Thus, the level of priority $\alpha$ for satisfying a preference is changed by the involutive mapping $1 - (\cdot)$ into a satisfaction level when this preference is violated.

**Example 1:** Let N be a CP-net over variables $X_1$, $X_2$ and $X_3$, let $\Gamma$ be a set of constraints, $\varphi_i \in \Gamma$, where $\varphi_1 = \top : n > \bar{n}$, $\varphi_2 = \top : o > \bar{o}$, $\varphi_3 = n : p > \bar{p}$, $\varphi_4 = o : p > \bar{p}$ and $\varphi_5 = \bar{n}\bar{o} : \bar{p} > p$. These constraints do not encode a complete CP-Net. But it can be completed by making it explicit with the additional constraints : $\varphi_6 = no : p > \bar{p}$, $\varphi_7 = n\bar{o} : p > \bar{p}$

and $\varphi_7 = \bar{n}o : p > \bar{p}$. Note that in possibilistic logic, we are not obliged to explicit all these constraints, indeed it is encoded by the possibilistic constraints $K_1 = \{c_1 = (n, \alpha), c_2 = (o, \beta), c_3 = (\bar{n} \vee p, \gamma), c_4 = (\bar{o} \vee p, \delta), c_5 = (n \vee o \vee \bar{p}, \varepsilon)\}$. Since the values of the weights $\alpha, \beta, \gamma, \delta, \varepsilon$ are unknown, no particular ordering is assumed between them. Table 1 gives the satisfaction levels for the possibilistic clauses encoding the five elementary preferences, and the eight possible choices. The last column gives the global satisfaction level by minimum combination.

**Table 1.** Possible alternative choices in Example1.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | min |
|---|---|---|---|---|---|---|
| nop | 1 | 1 | 1 | 1 | 1 | 1 |
| no$\bar{p}$ | 1 | 1 | 1-$\gamma$ | 1-$\delta$ | 1 | 1-$\gamma$,1-$\delta$ |
| n$\bar{o}$p | 1 | 1- $\beta$ | 1 | 1 | 1 | 1- $\beta$ |
| n$\bar{o}\bar{p}$ | 1 | 1- $\beta$ | 1-$\gamma$ | 1 | 1 | 1- $\beta$,1-$\gamma$ |
| $\bar{n}$op | 1- $\alpha$ | 1 | 1 | 1 | 1 | 1- $\alpha$ |
| $\bar{n}$o$\bar{p}$ | 1- $\alpha$ | 1 | 1 | 1-$\delta$ | 1 | 1- $\alpha$,1-$\delta$ |
| $\bar{n}\bar{o}$p | 1- $\alpha$ | 1- $\beta$ | 1 | 1 | 1-$\varepsilon$ | 1- $\alpha$, 1- $\beta$,1-$\varepsilon$ |
| $\bar{n}\bar{o}\bar{p}$ | 1- $\alpha$ | 1- $\beta$ | 1 | 1 | 1 | 1- $\alpha$ ,1- $\beta$ |

Even if the values of the weights are unknown, as it is the case in the above example, a partial order between the interpretations (they are 8 in our example) is naturally induced by a Pareto ordering (denoted $\succ_{Par}$) between the corresponding vectors evaluating the satisfaction levels with respect to the constraints.

Generally speaking, let $K = \{(a_i, \alpha_i)\}$ be a set of formulas associated with symbolic weights. Let $t, t'$ be two interpretations of the set of formulas $\{a_i | i = 1, n)\}$ associated with the vectors of their evaluations with respect to each formula in $K$. Then, we have

$$t \succ_{Par} t' \text{ iff } \Sigma_t \subset \Sigma_{t'},$$

where $\Sigma_t$ (resp. $\Sigma_{t'}$) is the set of formulas in $K$ violated by $t$ (resp. $t'$).

In our example, we have for instance the following $Pareto$ orderings between the 5-component vectors

$$(1-\alpha, 1, 1, 1, 1) \succ_{Par} (1-\alpha, 1-\beta, 1, 1, 1) \succ_{Par} (1-\alpha, 1-\beta, 1, 1, 1-\varepsilon)$$

whatever the values of $\alpha, \beta, \varepsilon$. Thus, we get the following partial order between interpretations:
$nop \succ_{Par} \{no\bar{p}, n\bar{o}p, \bar{n}op, n\bar{o}\bar{p}, \bar{n}o\bar{p}, \bar{n}\bar{o}\bar{p}, \bar{n}\bar{o}p\}$
$\bar{n}op \succ_{Par} \bar{n}\bar{o}p \succ_{Par} \bar{n}\bar{o}\bar{p}$
$n\bar{o}p \succ_{Par} n\bar{o}\bar{p}$ ; $\bar{n}op \succ_{Par} \bar{n}o\bar{p}$

Thus, this partial order amounts to rank-ordering a vector $v'$ after a vector $v$, each time the set of preferences violated in $v$ is strictly included in the set of preferences violated in $v'$, since nothing is known on the relative values of the symbolic levels (except they are strictly smaller than 1, when different from 1). Then a vector $v$ is greater than another $v'$, only when the components of $v$ are equal to 1 for those components that are different in $v$ and $v'$.

We could also use the $discrimin$ order denoted by $\succ_{discrimin}$ defined in the following way: identical vector components are discarded, and the minima of the remaining components for each vector are compared. Note that $t$ and $t'$ are comparable only if one of the two minima returns 1 (which is the only evaluation known to be greater than any symbolic weight ($\neq 1$)). In fact here, the orderings $\succ_{Par}$ and $\succ_{discrimin}$ coincide.

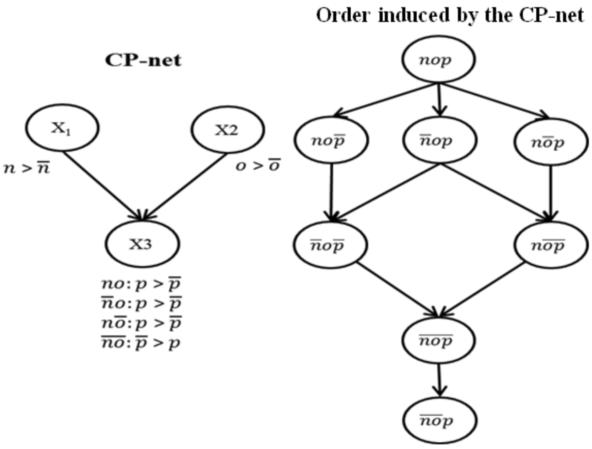


**Figure 1.** CP-net and partial order induced by it

## 4.3 Introducing preferences between symbolic weights

The authors of [32] have proposed an encoding of CP-nets by imposing a partial order between the symbolic weights of formulas. The partial order on symbolic weights is defined as follows. For each pair of formulas $(\neg u_i \vee x, \alpha_i)$ and $(\neg u_j \vee y, \alpha_j)$ such that $X$ is a father of $Y$ where $u_j$ is $(\neg u_i \vee \neg x)$ or $(\neg u_i \vee x)$, we put $\alpha_i > \alpha_j$ [32, 28]. These constraints between symbolic weights can be obtained by Algorithm 1, which computes the partial order between symbolic weights from a set of ceteris paribus statements.

Once we have got this partial order over symbolic weights, we use the *leximin* order defined below, for refining the $\succ_{Par}$ ordering used before:

**Leximin with partially ordered weights**: Let $\Psi = \{1 - \alpha_1, \cdots 1 - \alpha_n, 1\}$ be a set of symbolic possibility degrees , and $\omega, \omega'$ two interpretations $\in \Omega$. Let $\Psi(\omega) = (\pi_i(\omega) \cdots \pi_n(\omega)), \Psi(\omega') = (\pi_i(\omega') \cdots \pi_n(\omega'))$ be their vectors of evaluation in terms of symbolic weights (with respect to the violated formulas). Then the leximin ordering denoted $\succ_{lex}$ between vectors of values belonging to a totally ordered set consists in applying the discrimin procedure after reordering their components in increasing order. The leximin ordering can be extended as follows:

- delete all pairs $(\pi_i(\omega), \pi_j(\omega'))$ where $\pi_i(\omega) = \pi_j(\omega')$ so we get $\Psi^*(\omega)$ and $\Psi^*(\omega')$ where $\Psi^*(\omega) \cap \Psi^*(\omega') = \emptyset$
- $\omega \succ_{lex} \omega'$ iff $\min(\Psi^*(\omega) \cup \Psi^*(\omega')) \subseteq \Psi^*(\omega)$
- $\omega$ and $\omega$ are incomparable iff $\min(\Psi^*(\omega) \cup \Psi^*(\omega')) \not\subseteq \Psi^*(\omega)$ and $\min(\Psi^*(\omega) \cup \Psi^*(\omega')) \not\subseteq \Psi^*(\omega')$.

Note that this $leximin$ ordering is the same as $discrimin$ and $Pareto$ orderings, if weights are incomparable. When some weights are comparable, $discrimin$ and $Pareto$ orderings still coincide due to the particular nature of the vectors that are compared (i.e., vectors $(u_1, \cdots, u_i, \cdots, u_n)$ such as $u_i \in \{1, 1 - \alpha_i\}$), but the extended $leximin$ refines the Pareto ordering.

In Example 1, in the order induced by the Pareto ordering, the interpretations $n\bar{o}\bar{p}$, $\bar{n}o\bar{p}$, $\bar{n}\bar{o}\bar{p}$ are incomparable. Applying algorithm

1, we give priority to father nodes, i.e., here, we introduce the following constraints between the symbolic weights $\alpha > max(\gamma, \delta, \varepsilon)$ and $\beta > max(\gamma, \delta, \varepsilon)$. Then, the application of leximin ordering allows us to distinguish between $\{n\bar{o}\bar{p}, \bar{n}o\bar{p}\}$ and $\bar{n}\bar{o}\bar{p}$. So, the order induced by the CP-net, or equivalently the one induced by the possibilistic approach giving priority to father nodes (see Figure 1) is:

$nop \succ_{lex} \{no\bar{p}, n\bar{o}p, \bar{n}op\}$,
$\{no\bar{p}, \bar{n}op\} \succ_{lex} \bar{n}o\bar{p}$, $\{n\bar{o}p, \bar{n}op\} \succ_{lex} n\bar{o}\bar{p}$
$\{n\bar{o}\bar{p}, \bar{n}o\bar{p}\} \succ_{lex} \bar{n}\bar{o}\bar{p} \succ_{lex} \bar{n}\bar{o}p$

**Algorithm 1** calculates the relative importance between CP-net preferences statements

```
Require: C a set of constraints of the form (P_i, α_i)
  Γ a set of preference statement of the form u : x > x'
  IDC=∅
  for φ_i = u_i : x_i > x'_i in Γ do
    for c_j in C do
      if c_j is of the form (u_i, α_j) then
        for c_k in C do
          if c_k is of the form (¬u_i ∨ x_i, α_k) then
            IDC ← IDC + (α_j > α_k)
          end if
        end for
      end if
    end for
  end for
  return IDC
```

## 5 CP-THEORIES IN POSSIBILISTIC LOGIC

Wilson [35, 36] has proposed a new formalism named CP-theories that extends CP-nets and TCP-nets in order to express stronger conditional preferences as well as the usual CP-net ceteris paribus statements. For a set of variables $V$ , the language $\mathcal{L}_{\mathcal{V}}$ (abbreviated to $\mathcal{L}$) consists of all statements of the form $u : x > x'$ $[W]$, where $u$ is an assignment to a set of variables $U \subseteq$ V (i.e., $u \in \underline{U}$), $x, x' \in X$ are different assignments to some variable $X \notin U$ (and so $x$ and $x'$ correspond to different values of $X$) and $W$ is some subset of $V - U - \{X\}$. If $\varphi$ is the statement $u : x > x'$ $[W]$, we may write $u_\varphi = u, U_\varphi = U, x_\varphi = x, x'_\varphi = x', W_\varphi = W$ and $T_\varphi = V - (\{X\} \cup U \cup W)$. Subsets of $\mathcal{L}$ are called conditional preference theories or CP-Theories (on $V$). For $\varphi = u : x > x'$ $[W]$, let $\varphi^*$ be the set of pairs of interpretations $\{(tuxw, tux'w') : t \in \underline{T_\varphi}, w, w' \in \underline{W}\}$. Such pairs $(\omega, \omega') \in \varphi^*$ are intended to represent a preference for $\omega$ over $\omega'$, and $\varphi$ is intended as a compact representation of the preference information $\varphi^*$. Informally, $\varphi$ represents the statement that, given $u$ and any $t$, $x$ is preferred to $x'$, *irrespective of* the assignments to $W$, it means that we prefer any outcome with $x$ to any outcome with $x'$, in the context $u$. For conditional preference theory $\Gamma \subseteq \mathcal{L}$, define $\Gamma^* = \bigcup_{\varphi \in \Gamma} \varphi^*$, so $\Gamma^*$ represents a set of preferences. We assume here that preferences are transitive, so it is then natural to define order $>_\Gamma$ , induced on $V$ by $\Gamma$, to be the transitive closure of $\Gamma^*$. With this type of statements (CP-theory statements), we can represent a CP-net by a statement $u : x > x'$ $[W]$ with $W = \emptyset$ and a TCP-net with $W$ containing at most one variable [36].

In possibilistic logic, a CP-theory statement $\varphi = u : x > x'$ $[W]$ is represented by : $\Delta(tux) > \Pi(tux')$ standing for $min_{\omega \models tux} \pi(\omega) > max_{\omega' \models tux'} \pi(\omega')$ [33] which has the same semantics as the “irrespectively" constraint (given $u$ $x$ is preferred to $x'$ irrespective of the assignments to $W$). The possibilistic encoding of CP-theory expression uses exactly the same possibilistic formulas (with symbolic weights) as for the corresponding CP-net expression (when W is ignored). All the additional constraints between the weights of the father nodes with respect of child node are also maintained. Further, constraints between weights are added according to the procedure that we describe now.

Consider a CP-theory expression $u : x > x'$ $[W]$. It is encoded by a possibilistic preference statement $(\neg u \vee x, \alpha_i)$. Then we shall add the constraint $\alpha_i > \alpha_j$ for any $\alpha_j$, such that $(\neg u \vee w, \alpha_j)$ is a possibilistic preference statement, with the same context $u$, over one variable (or more) $w \in \underline{W}$. These constraints over weights can be obtained by Algorithm 2: from a set of CP-theory statements of the forme $u : x > x'[W]$, we elicit a partial order over symbolic weights used for inducing the same order between interpretations as the CP-theory. This procedure indeed guarantees that the constraints of the form $\Delta(tux) > \Pi(tux')$ which is same as $\forall w, w' \in \underline{W}, \pi(tuxw) > \pi(tux'w')$ will be satisfied. Let us give a sketch of the reason why:

Consider $\underline{X} = \{x, x'\}$ and $\underline{W} = \{w, w'\}$, the possibilistic encoding of the constraint will be $c_i = (\neg u \vee x, \alpha_i)$, and consider that we got a possibilistic constraint $c_j = (\neg u \vee w, \alpha_j)$. Let the possibility distribution of the constraint $\Delta(tux) > \Pi(tux')$ $\forall t \in T$ :

- $\pi(tuxw) > \pi(tux'w)$
- $\pi(tuxw') > \pi(tux'w)$
- $\pi(tuxw) > \pi(tux'w')$
- $\pi(tuxw') > \pi(tux'w')$

**Proof**: we proceed using reductio ad absurdum, so, we suppose that $\alpha_j \geq \alpha_i$. Consider the two interpretations $\omega_1 = tuxw'$ and $\omega_2 = tux'w$, $\omega_1$ satisfies the first constraint ($c_i$) and falsifies the second one ($c_j$), however, $\omega_2$ falsifies the first constraint and satisfies the second one, let $v_1 = (1, 1 - \alpha_j)$ and $v_2 = (1 - \alpha_i, 1)$ be the vectors of satisfactions associated to $\omega_1$ and $\omega_2$ respectively, $\omega_1 \succ \omega_2$ imply $1 - \alpha_j > 1 - \alpha_i$, that means $\alpha_j < \alpha_i$ (contradiction) QED.

**Example 2 [36] :** Let $\Gamma$ be a CP-Theory over three variables $X_1, X_2$ and $X_3$, composed of set of preferences statements $\varphi_{1-5}$ given by: $\varphi_1$ = $\top : x_1 > \bar{x_1}[X_2, X_3]$, $\varphi_2$ = $x_1 : x_3 > \bar{x_3}[X_2]$, $\varphi_3$ = $x_1 : x_2 > \bar{x_2}$, $\varphi_4$ = $\bar{x_1} : x_2 > \bar{x_2}[X_3]$, $\varphi_5$ = $\bar{x_1} : x_3 > \bar{x_3}$, this statements are coded in possibilistic logic by:
$K_2 = \{c_1 = (x_1, \alpha), c_2 = (\bar{x_1} \vee x_3, \beta), c_3 = (\bar{x_1} \vee x_2, \gamma), c_4 = (x_1 \vee x_2, \delta), c_5 = (x_1 \vee \bar{x_3}, \varepsilon)\}$.
Table 2 gives the satisfaction levels for the possibilistic clauses encoding the five elementary preferences, and the eight possible choices. The last column gives the global satisfaction level by minimum combination.

After applying the *Pareto* ordering (or equivalently here, *discrimin* ordering), what we get is an ordering which is less refined than the ordering induced by the CP-theory or by the CP-net (see Figure 2). But we can capture the CP-theory ordering by taking into account an ordering between weights that reflects the relative importance of the constraints, and which can be elicited from the CP-theory. In the example, we should enforce $\alpha > max(\beta, \gamma, \delta, \varepsilon)$ due CP-net “father" constraints ($X_1$ is the father of $X_2$ and of $X_3$);

**Table 2.** Possible alternative choices in Example2.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | min |
|---|---|---|---|---|---|---|
| $x_1x_2x_3$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $x_1x_2\bar{x_3}$ | 1 | 1-$\beta$ | 1 | 1 | 1 | 1-$\beta$ |
| $x_1\bar{x_2}x_3$ | 1 | 1 | 1-$\gamma$ | 1 | 1 | 1-$\gamma$ |
| $x_1\bar{x_2}\bar{x_3}$ | 1 | 1-$\beta$ | 1-$\gamma$ | 1 | 1 | 1-$\beta$,1-$\gamma$ |
| $\bar{x_1}x_2x_3$ | 1-$\alpha$ | 1 | 1 | 1 | 1-$\varepsilon$ | 1-$\alpha$,1-$\varepsilon$ |
| $\bar{x_1}x_2\bar{x_3}$ | 1-$\alpha$ | 1 | 1 | 1 | 1 | 1-$\alpha$ |
| $\bar{x_1}\bar{x_2}x_3$ | 1-$\alpha$ | 1 | 1 | 1-$\delta$ | 1-$\varepsilon$ | 1-$\alpha$,1-$\delta$,1-$\varepsilon$ |
| $\bar{x_1}\bar{x_2}\bar{x_3}$ | 1-$\alpha$ | 1 | 1 | 1-$\delta$ | 1 | 1-$\alpha$,1-$\varepsilon$ |

besides, we have $\beta > \gamma$ due to the "irrespectively" constraint [w. r. t. $X_2$] in $\varphi_2$ and we have $\delta > \varepsilon$ due to the "irrespectively" constraint [w. r. t. $X_3$] in $\varphi_4$ (by applying the procedure explained above, or Algorithm 2). Then, the order induced by the CP-theory and the one captured by the possibilistic approach (taking account the above inequalities between symbolic weights) coincide. It is given by: $x_1x_2x_3 \succ_{lex} x_1\bar{x_2}x_3 \succ_{lex} x_1x_2\bar{x_3} \succ_{lex} x_1\bar{x_2}\bar{x_3} \succ_{lex} \bar{x_1}x_2x_3 \succ_{lex} \bar{x_1}x_2x_3 \succ_{lex} \bar{x_1}\bar{x_2}x_3 \succ_{lex} \bar{x_1}x_2\bar{x_3}$

**Algorithm 2** calculates the relative importance between CP-theory preferences statements

**Require:** C a set of constraints of the form ($P_i, \alpha_i$)
$\Gamma$ a set of preference statement of the form $u: \ x > x'[W]$
IDC=$\emptyset$
**for** $\varphi_i = u_i : x_i > x'_i[W_i]$ in $\Gamma$ **do**
  **if** $W_i$=$\emptyset$ **then**
    IDC $\leftarrow$ IDC + **Algorithm 1** (C ,$\{\varphi_i\}$)
  **else**
    **for** $c_j$ in C **do**
      **if** $c_j$ is of the form $(\neg u_i \vee x_i, \alpha_j)$ **then**
        **for** $c_k$ in C **do**
          **if** $c_k$ is of the form $(\neg u_i \vee \neg x_i \vee v, \alpha_k)$ or $(\neg u_i \vee z, \alpha_k)/z \in \underline{W_i}, v \in \{\underline{V-U}\}$ **then**
            IDC $\leftarrow$ IDC + $(\alpha_j > \alpha_k)$
          **end if**
        **end for**
      **end if**
    **end for**
  **end if**
**end for**
**return** IDC

As a summary, the Pareto ordering (here equivalent to the discrimin ordering) is obtained without introducing any inequality constraint between importance weights (all symbolic weights, distinct from 1, remain incomparable). Then the CP-net is obtained by enforcing priorities in favor of constraints associated with "father" nodes, but ignoring the "irrespectively" constraints of the CP-theory. Note that Pareto ordering is compatible with the CP-net and CP-theory orderings, but less refined, and the CP-net ordering is less refined than the CP-theory one (due to the ignorance of "irrespectively" constraints).

## 6 CONCLUDING REMARKS

In this paper, the possibilistic logic framework has been recalled and its interest for preference representation strongly advocated. Clearly, possibilistic logic is still close to classical logic, but the introduction

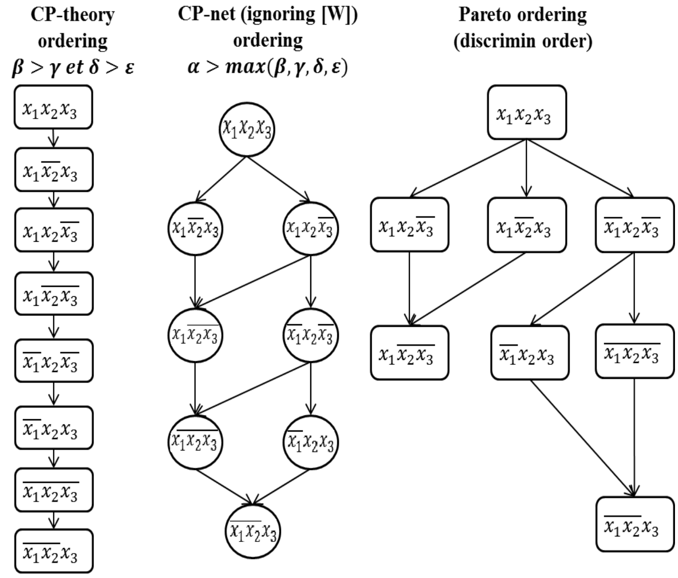


**Figure 2.** Lexmin, Cp-net and CP-Theory orders in Example 2

of weights substantially increases its representation capabilities, especially with respect to inconsistency handling. We have shown how the use of symbolic weights in the possibilistic logic setting enables us to deal with partial orders (encoding CP-nets and CP-theories in this way). This constitutes an alternative to the introduction of a preference relation inside the representation language, as in, e.g., [12].

Moreover, it has been recalled how the use of symbolic weights [11] enables us to represent CP-nets faithfully in the possibilistic logic setting, by imposing greater priority weights to father nodes. Moreover, possibilistic logic with symbolic weights has a representation power much richer than the one of CP-nets, since, e.g., one may give priority to a constraint associated with a child node (which is impossible in CP-nets or in TCP-nets). Then, after restating the CP-theory representation framework, and results illustrating its expressive power which generalizes CP-nets and TCP-nets [36], we have shown that a CP-theory can be faithfully represented in possibilistic logic by introducing further inequalities between symbolic weights in order to take into account the CP-theory idea that some preferences hold irrespective of the values of some variables. An interesting question for further research would be to examine the possible relations that may exist between the non symmetrical notion of independence in possibilistic networks [1] and some limitations of graphical representation settings such as CP-nets.

We have also indicated that our handling of preferences statements in the style of Qualitative Choice Logic remains close to mainstream database approaches to preference queries pioneered by Lacroix and Lavency [34]. It has also already pointed out that Chomicki's approach [16] based on winnow operator can be also expressed in our setting [28].

Lastly, let us also mention other possibilistic logic-based works in preference modeling where one may handle both general statements about importance levels and (couter)-examples [19, 26]. This kind of approach may also incorporate a Choquet's integral-like handling of importance levels [31]. Moreover, a possibilistic logic representation of Sugeno integral has been recently proposed [25], and last but not least possibility theory setting enables to represent bipolar preferences, where both negative preferences (rejections) and positive preferences (what is really desired) can be expressed [7].

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